DISCRETE MODE MATCHING FOR THE FULL-WAVE ANALYSIS OF PLANAR WAVEGUIDE STRUCTURES

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ABSTRACT

A new spectral domain method is proposed for the exact and efficient numerical full-wave analysis of planar multilayer and multiconductor optical and microwave waveguide structures. It includes a systematic procedure with very less analytical and computational effort. The propagation constants are calculated for some examples and compared to existing results.

INTRODUCTION

One of the most widely applied methods for the analysis of planar waveguide structures and resonators is the spectral domain approach (SDA) which can be used for open as well as for enclosed structures [1], [2]. However, the application requires a significant analytical preprocessing in formulating the integral equations involving the dyadic Green's function of stratified dielectric layers in spectral domain and in choosing suitable basis functions for the current components on the metallizations in the interfaces. The evaluation of the integrals makes a discussion of the integration path and the location of the surface wave poles of the Green's function in the complex plane necessary.

Another method, which is simpler to apply and therefore has found growing interest in recent years, is the method of lines (MoL). It uses a finite difference formulation for the differential operators and a discrete Fourier transform of the field components [3], [4]. It has turned out, however, that the approximation of operators and eigenvalues causes an error that increases drastically for higher order modes and leads to a disadvantageous convergence behavior requiring large storage and unnecessary long computation time for acceptable results [6].

A first attempt has been made to combine MoL and mode-matching to the space-spectral-domain approach (SSDA) [5]. But this procedure is also based on the approximation of the differential operators.

It is the aim of this contribution to introduce a new procedure that combines the advantages of both methods but mostly avoids their shortcomings. All field and current components are represented by an orthogonal set of basis functions which are the eigensolutions of Helmholtz' wave equation for suitable boundary conditions. Corresponding to the discrete Fourier transform, these functions are discretized at equidistant points of which the number is related to the highest order mode in accordance with the sampling theorem.

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FOUNDATIONS AND APPLICATION

A typical planar waveguide structure is shown in Fig. 1. It may consist of multiple dielectric layers with multiple conducting strips in the interfaces of arbitrary layers. Starting point of the analysis is the wave equation (normalized with k_0)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \varepsilon_r - k_z^2\right)\psi_{e,h} = 0 \qquad (1)$$

for the two independent field components $\psi_{e,h} = E_z, H_z$ and an assumed wave propagation

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$\exp j(wt - k_z z)$ within any layer enclosed by lateral electric and/or magnetic walls. E_z and H_z must then fulfill different boundary conditions which are of the Dirichlet and Neumann type.

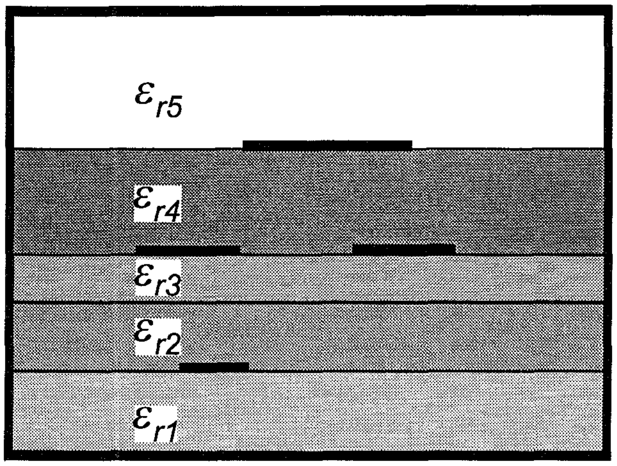

Fig. 1. Multilayer/multiconductor microwave structure

Solving the complete boundary value problem by means of a modal expansion of the fields given by the Fourier series

$$\psi(x,y) = \sum_{i=1}^{\infty} \tilde{\psi}_i(y) \sin(k_{xi}x + \varphi_i) \qquad (2)$$

with k_{xi} and φ_i following from the corresponding boundary condition, results in a set of ordinary differential equations

$$\left(\boldsymbol{I}\frac{d^2}{dy^2} - \boldsymbol{k}_y^2 \right) \tilde{\Psi} = \mathbf{0} \qquad (3)$$

with the diagonal matrix $\boldsymbol{k}_y^2 = \boldsymbol{k}_x^2 - (\varepsilon_r - k_z^2)\boldsymbol{I}$.

Writing (2) as the scalar product $\Psi_{e,h} = \tilde{\Psi}_{e,h}\mathbf{t}_{e,h}$ with the modal vectors $\mathbf{t}_{e,h}$, we always find that $\frac{\partial}{\partial x}\mathbf{t}_e = \mathbf{t}_h$ and $\frac{\partial}{\partial x}\mathbf{t}_h = -\mathbf{t}_e$ or vice versa which is a consequence of the dual boundary conditions.

Using this feature and the orthogonality property of trigonometric functions, we are able to transform all field components into spectral domain and with the solution of (3) a hybrid matrix formulation is obtained combining the tangential field components on both sides of an arbitrary layer i (Fig. 2)

$$\begin{bmatrix} \tilde{\mathbf{E}}_i \\ \tilde{\mathbf{H}}_i \end{bmatrix} = \widetilde{\boldsymbol{K}}_i \begin{bmatrix} \tilde{\mathbf{E}}_{i-1} \\ \tilde{\mathbf{H}}_{i-1} \end{bmatrix} \qquad (4)$$

with the special definitions $\tilde{\mathbf{E}}_i = \begin{bmatrix} \tilde{\mathbf{E}}_x \\ -j\tilde{\mathbf{E}}_z \end{bmatrix}_i$, $\tilde{\mathbf{H}}_i = \eta_0 \begin{bmatrix} -j\tilde{\mathbf{H}}_z \\ \tilde{\mathbf{H}}_x \end{bmatrix}_i$.

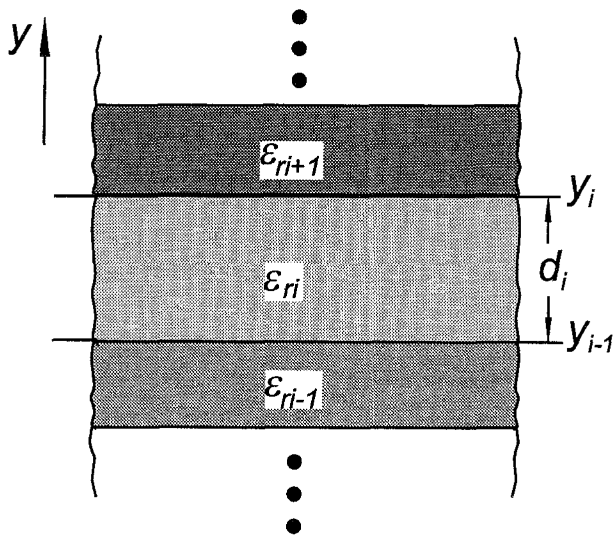

Fig. 2. Stratified dielectric

For the whole stratified structure an equivalent circuit can now be given (Fig. 3). The metallizations in the interfaces are represented by current sources $\tilde{\mathbf{J}} = \eta_0 \begin{bmatrix} j\tilde{\mathbf{J}}_x \\ \tilde{\mathbf{J}}_z \end{bmatrix}$ and thus all continuity requirements are fulfilled automatically. For open structures, the admittance matrices $\widetilde{\boldsymbol{Y}}_{0,n}$ for the termination at the bottom or the top include the radiation conditions. In the case of metallizations they have to be replaced by electric shorts $\left(\tilde{\mathbf{E}}_{0,n} = \mathbf{0}\right)$.

What follows is simple network analysis technique to obtain an equation combining the currents on the strips and the tangential electric field components in the interfaces by means of

$$\tilde{\boldsymbol{Z}} \cdot \tilde{\mathbf{J}} = \tilde{\mathbf{E}}\,. \qquad (5)$$

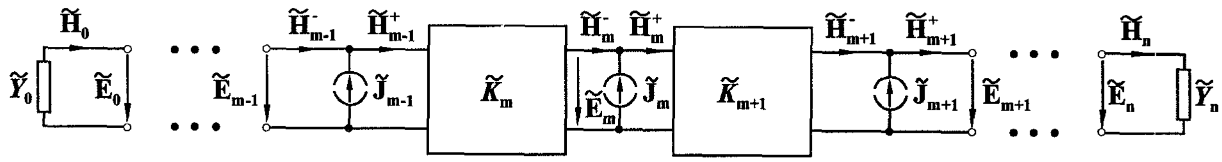

Fig. 3. Combined TE/TM equivalent circuit for the whole structure

Herein, $\tilde{Z}$ is known as electric field dyadic Green's function in spectral domain. It's general form for a structure with multiple metallizations in the interfaces $m = 1, \cdots, M$ is given by (cf. [7])

$$\tilde{Z}^{-1} = \begin{bmatrix} \tilde{L}_{11} & \tilde{L}_{12} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \tilde{L}_{21} & \tilde{L}_{22} & \tilde{L}_{23} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{L}_{32} & \tilde{L}_{33} & \tilde{L}_{34} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \tilde{L}_{43} & \tilde{L}_{44} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \tilde{L}_{MM} \end{bmatrix} . \quad (6)$$

The preceding steps are similar to the approach described in detail in [7]. They can be performed numerically if the structure is complicated or analytically for simple configurations. In the latter case, the analysis takes place in Fourier transform domain and the spectral wave number k_x is simply replaced by its discrete form, the diagonal matrix $\boldsymbol{k}_x$.

For a solution of (5), the boundary condition $\mathbf{E}_{\tan} = \mathbf{0}$ on the metallic strips has to be met. To this end, field and current components are taken only at equidistant points $x_j = h \cdot j$, $j = 1, 2, \cdots, N$ and the Fourier series is cut at $i = N$ according to the sampling theorem. This gives the procedure its name as discrete mode matching (DMM). Under these conditions, the discretized components are obtained by means of a transformation $\Psi = T\tilde{\Psi}$ with the modal matrix T combined of the column vectors $\mathbf{t}$ at the discretization points. It is identical to the transformation matrix used in the MoL and has been proven to be orthogonal because it is related to the DFT. Since $\mathbf{J} = \mathbf{0}$ outside the strip, we obtain a reduced set of equations in space domain

$$Z_{\text{red}}\mathbf{J} = \mathbf{0} \quad (7)$$

which is solved as an indirect eigenvalue problem for the propagation constant k_z.

RESULTS AND CONCLUSION

For the example of the propagation constant of a partially filled waveguide, the accuracy of the DMM is compared to the MoL and depicted in Fig. 4. With the DMM we directly obtain the analytical solution for each TMy-mode according to the sampling theorem, whereas in the MoL a large oversampling rate is necessary due to the approximation of operators and eigenvalues.

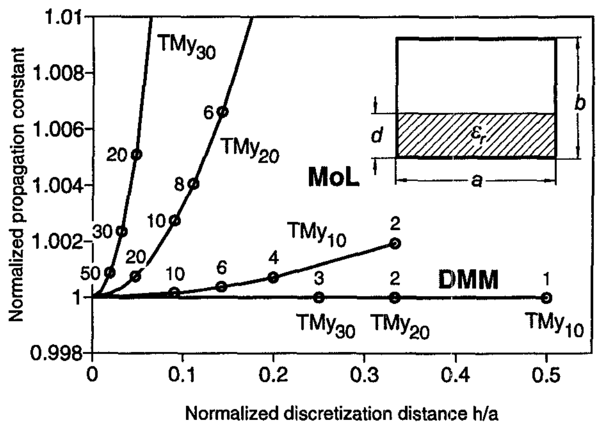

Fig. 4. Convergence of the normalized propagation constants of the fundamental and higher order modes of a partially filled waveguide to their exact analytical values. $a/\lambda_0 = 2$, $b/a = 0.4$, $d/b = 0.2$, $\varepsilon_r = 2.55$. MoL: Method of lines, DMM: Discrete mode matching. The data labels indicate the number of discretization points for one field component.

The next example shows the variety of this method and its applicability to arbitrary complicated structures (Fig. 5). In principle there are no restrictions of the number of dielectric layers and microstrip lines.

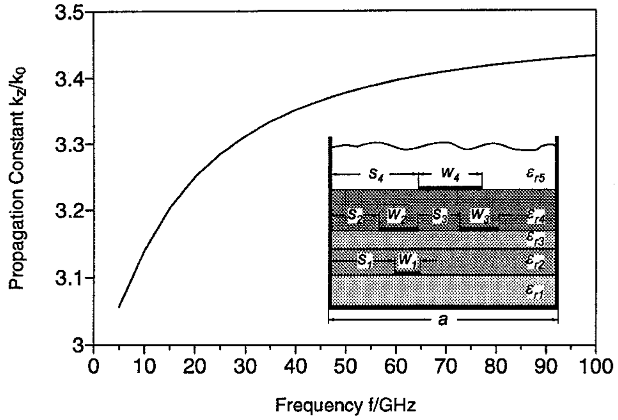

Fig. 5. Dispersion characteristic of the fundamental mode of a multilayer/multiconductor microwave structure (open at the top) over a wide frequency range. $a = 20\text{mm}, w_1 = 1.1\text{ mm}, w_2 = 1.6\text{ mm}, w_3 = 1.8\text{ mm}, w_4 = 2.3\text{ mm}, s_1 = 8.7\text{mm}, s_2 = 7.5\text{mm}, s_3 = 1.6\text{mm}, s_4 = 9.2\text{mm}, d_1 = 1.2\text{mm}, d_2 = 1.0\text{mm}, d_3 = 0.7\text{mm}, d_4 = 1.5\text{mm}, \varepsilon_{r1} = 2.3, \varepsilon_{r2} = 8.875, \varepsilon_{r3} = 5.0, \varepsilon_{r4} = 12.0, \varepsilon_{r5} = 1.0$

The generalization of this procedure to inhomogeneous dielectrics, finite thicknesses and conductor losses as well as to 3 dimensions and other coordinate systems is straightforward and will determine future works. It is also possible to include lateral absorbing boundaries for the investigation of radiation effects.

To sum up, the DMM provides an easy to apply CAD procedure with minimal discretization and nonoscillating convergence behavior, which should be very advantageous for the analysis and design of planar optical and microwave waveguide structures.

REFERENCES

[1] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496-499, July 1973.

[2] T. Itoh and W. Menzel, "A full-wave analysis method for open microstrip structures," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 63-68, Jan. 1981.

[3] R. Pregla and W. Pascher, "The Method of Lines," in *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, T. Itoh, Ed. New York: Wiley, 1989.

[4] A. Dreher and R. Pregla, "Full-wave analysis of radiating planar resonators with the method of lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-41, pp. 1363-1368, Aug. 1993.

[5] M. Yu and R. Vahldieck, "On the nature and application of the space-spectral-domain approach (SSDA)," in *1994 IEEE Int. Microwave Symp.*, pp. 591-594.

[6] A. Dreher and T. Rother, "New aspects of the method of lines," *IEEE Microwave Guided Wave Lett.*, vol. 5, pp. 408-410, Nov. 1995.

[7] A. Dreher, "A new approach to dyadic Green's function in spectral domain," *IEEE Trans. Antennas Propagat*, vol. AP-43, pp. 1297-1302, Nov. 1995.